

Compliant Control of an Upper Limb Rehabilitation Robot Based on Impedance Design

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Abstract. Robot-assisted therapy is an essential rehabilitation method to regain motor function in upper limb impairment after neurological events such as stroke. The most challenging and problematic issue in this field is to realize safe and natural human-robot interaction. Conventional robots, which are generally controlled by rigid position controllers, cannot be compliantly adjusted according to the patient's voluntary movement or spasm. The appearance of high interaction forces is also a potential cause of secondary damage. This is a big research gap that requires further research on advanced compliant control. This paper is devoted to addressing this problem by developing an adaptive impedance controller to realize compliant control for an upper limb rehabilitation robot. The dynamic model of the robot is established in the paper, and the adaptive impedance control law is formulated. Then the parameters of the impedance controller can be adjusted in real-time based on interaction forces. To validate the effectiveness of the proposed controller, several simulation analyses under different scenarios are conducted, such as tracking a predefined trajectory without any external interference and with interaction forces generated by simulated patients. The simulation results indicate that besides the controller can guaranteeing stable and accurate tracking, the controller also has a good compliance performance, i.e., the interaction forces decrease significantly when external disturbances from patients appear. The effectiveness and superiority of the adaptive impedance control design are verified by the simulation results. The physical rehabilitation system based on the adaptive impedance control design has a good theoretical reference and simulation basis.

Keywords: Upper Limb Rehabilitation Robotics, Adaptive Impedance Control, Impedance Control.

1. Introduction

As the world's ageing accelerates, the rate of occurrence of upper limb motor dysfunction due to neurological e.g., stroke disorders, is rising each year, to a great burden on healthcare systems and society [1]. Thus, the need to deliver high-quality and long-term rehabilitation services is acute than ever before.

Manual therapy, as the traditional treatment method, has many disadvantages: it is rather labor-intensive, not easy to practice at a certain level of consistency and repeatability in training, and does not offer objective and quantitative ways to assess the patient's progress. Robot-assisted therapy has been developed as one of the possible solutions in this respect. They successfully induce neural plasticity as well as motor recovery through programmed, high-intensity, repetitive, and personalized training with rehabilitation robots [2].

Nonetheless, unlike the industrial robots that are used in a structured setting, the rehabilitation robots are subjected to large demands of safety, compliance, and interactivity, which involve direct human-robot interaction (HRI) [3]. Nevertheless, safety, compliance, and interactivity are exceptionally high in this type of application scenario. The most important factor is safety; there should be a system to avoid secondary harm to the patient, particularly when something is happening unexpectedly, such as muscle spasms. The compliance demands that the robot will adapt flexibly to the movements of the patient, similarly to the hands of a therapist. At the same time, interactivity plays a vital role in encouraging active patients who play a crucial role in the successful outcome of the neurorehabilitation process.

Hence, the focal technology that defines the clinical effectiveness of a rehabilitation robot is the control strategy [1]. Conventional position-based control measures that include Proportional-Integral-Derivative (PID) control can be used to obtain an accurate positioning. Nevertheless, they are often highly rigid. This strictness is unproductive and even harmful since it might contradict the voluntary actions of the user and cause pain or harm.

As a more effective way to deal with the issue of rigidity, compliant control emerged as the major area of research in the field of rehabilitation robotics. It aims at controlling the moving interaction between the position of the robot and the force of interaction. One of the mainstream strategies of attaining this objective is impedance control. It is the method by which the dynamic response of the robot to perturbations in position is controlled, making the robot appear to be a virtual mass-spring-damper system. With the help of the parameters of the impedance (i.e., the stiffness and damping), the controller may flexibly vary the softness or the stiffness of the robot. This renders it a perfect model of various rehabilitation steps, such as passive instructions to patients in a severe impairment state, to resistive exercise for individuals in more advanced recuperation periods.

2. Dynamic Modeling of the Rehabilitation Robot

The design and simulation of any sophisticated control system is a requirement in the presence of a definite mathematical representation of the robot. The model is the basis of studying the robot motion, the connection of the joint movement and the end-effector in behaving, and finally applying the compliant controller. This part provides the development of the kinematic model of the upper limb rehabilitation robot and finally gives the derivation of the Jacobian matrix needed in the analysis of velocity and force during impedance control.

2.1. Forward Kinematics Modeling

The robot under discussion is a 3-DOF anthropomorphic manipulator that has similar main movements to a human arm. The configuration of the manipulator is a waist joint, which is responsible for horizontal movement, a shoulder joint, which is responsible for flexion or extension movement in the vertical direction, and the elbow joint, which is responsible for flexion/extension movement in the vertical plane. The main aim of kinematic modeling is determination of forward kinematics equations which are equations describing Cartesian position of end-effector $P = [x, y, z]^T$ as a function of joint angles $q = [\theta_1, \theta_2, \theta_3]^T$. Manipulator's components are: Joint1(θ_1): waist joint, mounted on base, rotates around vertical Z-axis. Joint 2 (θ_2): shoulder joint, allows upper arm to move in vertical plane. Link1(L_1): upper arm link, connects shoulder to elbow. Joint3(θ_3): elbow joint, allows forearm to move with respect to upper arm Link2(L_2): forearm link, connects elbow to the end-effector

$$x = (L_2 \cos(\theta_2) + L_3 \cos(\theta_2 + \theta_3)) \cos(\theta_1) \quad (1)$$

$$y = (L_2 \cos(\theta_2) + L_3 \cos(\theta_2 + \theta_3)) \sin(\theta_1) \quad (2)$$

$$z = L_1 + L_2 \sin(\theta_2) + L_3 \sin(\theta_2 + \theta_3) \quad (3)$$

These three equations constitute the complete forward kinematic model of the robot, allowing for the computation of the end-effector's precise location in space given any set of joint angles.

2.2. Jacobian Matrix Derivation

For control design, especially for implementing impedance control, understanding the relationship between joint velocities and end-effector velocity is crucial. This relationship is linearly mapped by the Jacobian matrix, $J(q)$. The Jacobian relates the joint space velocity vector $\dot{q}$ to the task space velocity vector $\dot{p} = [v_x, v_y, v_z]^T$ through the fundamental equation $\dot{p} = J(q)\dot{q}$. It is formally

defined as the matrix of partial derivatives of the forward kinematic equations with respect to the joint variables:

$$J = \begin{bmatrix} \frac{\partial x}{\partial \theta_1} & \frac{\partial x}{\partial \theta_2} & \frac{\partial x}{\partial \theta_3} \\ \frac{\partial y}{\partial \theta_1} & \frac{\partial y}{\partial \theta_2} & \frac{\partial y}{\partial \theta_3} \\ \frac{\partial z}{\partial \theta_1} & \frac{\partial z}{\partial \theta_2} & \frac{\partial z}{\partial \theta_3} \end{bmatrix} \quad (4)$$

$s_i = \sin(\theta_i)$, $c_i = \cos(\theta_i)$, $s_{ij} = \sin(\theta_i + \theta_j)$. Assembling the partial derivatives yields the complete 3x3 Jacobian matrix:

$$J = \begin{bmatrix} -y & (-L_2 s_2 - L_3 s_{23})c_1 & -L_3 s_{23}c_1 \\ x & (-L_2 s_2 - L_3 s_{23})s_1 & -L_3 s_{23}s_1 \\ 0 & L_2 c_2 + L_3 c_{23} & L_3 c_{23} \end{bmatrix} \quad (5)$$

This Jacobian matrix is a fundamental tool for control design. It not only maps velocities but also allows for the transformation of forces between the task space and joint space via the relationship $\tau = J^T(q)F$, where τ is the vector of joint torques and F is the force applied at the end-effector. This transformation is central to the implementation of the impedance controller discussed in the subsequent section.

3. Impedance Control

3.1. Traditional Impedance Control

3.1.1 Mathematical model and control implementation of impedance control

The primary objective of impedance control is not to force the robot to strictly track a trajectory. Instead, its goal is to ensure that the end-effector, when interacting with the environment, behaves like a desired second-order system (i.e., a virtual mass-spring-damping system). This desired dynamic relationship is described by the following equation:

$$M_d(\ddot{x}_d - \ddot{x}) + B_d(\dot{x}_d - \dot{x}) + K_d(x_d - x) = F_{\text{ext}} \quad (6)$$

F_{ext} is the external environmental force perceived by the robot's end-effector. M_d , B_d , and K_d are the desired inertia, damping, and stiffness matrices set by the controller. $\ddot{x}_d$, $\dot{x}_d$, and x_d are the desired acceleration, velocity, and position, respectively. $\ddot{x}$, $\dot{x}$, and x are the actual acceleration, velocity, and position, respectively. The physical meaning of this equation is that the error in the robot's end-effector position ($e = x_d - x$) should be proportional to the external force F_{ext} it experiences.

This proportional relationship is dictated by the virtual spring-damping characteristics (K_d, B_d) defined in the control law. To make the robot exhibit this desired dynamic behavior, the controller must compute the corresponding joint torques. This is achieved by first calculating a command acceleration:

$$\dot{x}_{\text{cmd}} = \dot{x}_d + M_d^{-1}(B_d \dot{e} + K_d e - F_{\text{ext}}) \quad (7)$$

After obtaining this command acceleration, the controller then uses the robot's inverse dynamics model to calculate the necessary joint torques required to achieve this acceleration.

3.1.2 Simulation and analysis

To evaluate the system's dynamic performance, the robot end-effector was commanded to track a preset trajectory. An external force was applied at $t = 2s$ to observe the robot's compliant deviation and recovery. For the baseline (low-gain) parameter set (Fig. 1), the controller has insufficient dynamic stiffness and poor trajectory-tracking robustness. When hit by the external force, the system deviates significantly from the desired trajectory. After the disturbance is removed, the recovery

shows poor convergence with a long settling time, large overshoot, and notable oscillations, indicating limited disturbance-rejection capabilities.

In contrast, Fig. 2 shows the dynamic response of the optimized (high-gain) parameter set. It has high-performance trajectory tracking and excellent disturbance rejection. First, it has good transient performance, quickly suppressing an initial Z-axis position error of about -0.75m in 1.5 seconds for high-precision tracking. When the external force is applied at $t = 2$ s, the controller shows high dynamic stiffness. The instantaneous tracking error is suppressed to a minimum, peaking at -0.04m. The system has a very short settling time, converging within 1 second (around $t = 2.8$ s) smoothly without significant overshoot. This superior performance is due to the high-gain, high-bandwidth control law that generates high-amplitude, high-slew-rate torque to suppress position errors.

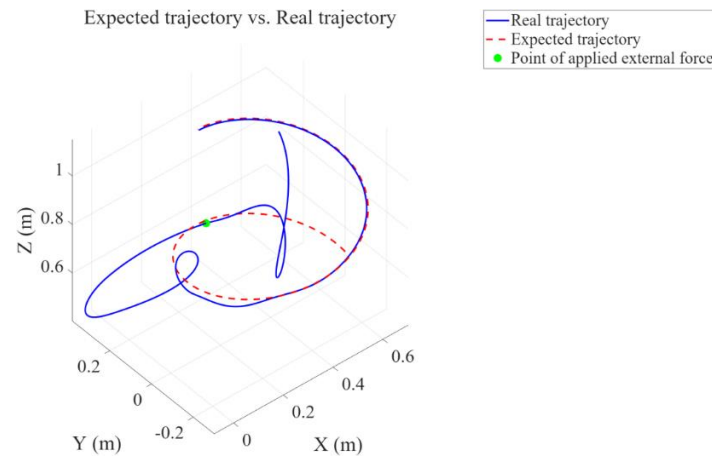


Fig. 1 Comparison of expected trajectory and actual trajectory after external force under low stiffness (Original)

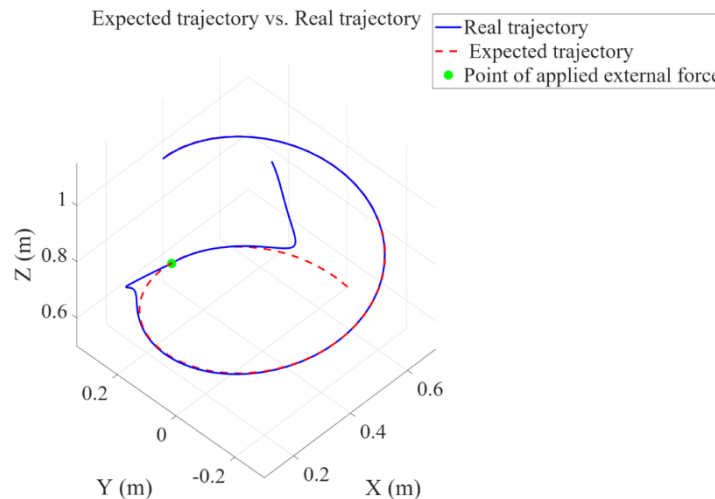


Fig. 2 Comparison of expected trajectory and actual trajectory after external force under high stiffness (Original)

3.2. Adaptive Impedance Control

3.2.1 Principles of adaptive impedance control

Traditional Fixed-Impedance Control imparts a constant compliance to the robot, which proves inadequate when facing dynamic and varied tasks. For instance, in a complete task chain involving "contact-operation-disengagement," the robot must possess high stiffness in free space to achieve fast and precise trajectory tracking. Conversely, upon contacting the environment (constrained space), it must rapidly switch to low stiffness to ensure compliant interaction and safety, preventing excessive contact forces from damaging the workpiece or the robot itself.

Adaptive Impedance Control is aimed to resolve this underlying paradox. It would seek to empower the robot to online adaptively adjust its impedance summers on the basis of online, real-time, perceived task surroundings as well as interaction conditions. The challenge is to achieve the advanced capability of a human hand, i.e. stiff when strength and accuracy is demanded, and soft when touching and motion is demanded.

Concisely, this technique is adaptive impedance control, which takes an important signal, the magnitude of the task error (positional or force-based), as a trigger to dynamically and smoothly alternate between a high-stiffness and high-precision mode and a low-stiffness and low-compliance mode.

3.2.2 Mathematical implementation: exponential decay stiffness mapping

The key and difficult issue in the stiffness switching process is to prevent sudden changes in parameters. When the $K_{\max}$ $K_{\min}$ stiffness value presents a step-wise variation at a certain instant, the dynamic properties of the system will be instantly changed. The control output chattering, mechanical vibration and system instability will be induced. Therefore, the stiffness mapping function should be designed to be continuous and smooth. The exponential decay function has been widely used due to its simple expression, fast computation and simple tuning.

Taking virtual stiffness K as an example, its adaptive regulation formula is designed as $K_d(e) = K_{d,\min} + (K_{d,\max} - K_{d,\min}) \cdot e^{-\alpha \|e\|}$ follows: $K_d(e)$ is the current stiffness value updated in real-time. $K_{\min}$ is the lower bound of stiffness, ensuring basic support and tracking capability. $K_{\max}$ is the upper bound for high-precision tasks. (e) is the $\|e\|$ error norm, quantifying the robot's deviation. $\|e\|$ It can be defined as position or $\|e_p\| = \sqrt{\tilde{x}^2 + \tilde{y}^2 + \tilde{z}^2}$ force error norm. ($\|e_F\| = \|F_{des} - F_{ext}\|$) α is the sensitivity factor regulating the stiffness response speed.

This formula achieves a smooth stiffness transition. When the error ($\|e\| \approx 0$), the exponential term $e^{-\alpha \|e\|} \approx e^0 = 1$, $K \approx K_{\min} + (K_{\max} - K_{\min}) \cdot 1 = K_{\max}$, and the system switches to maximum stiffness for trajectory accuracy. When the error is large $\|e\| \rightarrow \infty$, the exponential term $e^{-\alpha \|e\|} \rightarrow 0$, $K \approx K_{\min} + (K_{\max} - K_{\min}) \cdot 0 = K_{\min}$ approaches 0, and the stiffness decreases to the preset minimum for interaction safety. The exponential term $e^{-\alpha \|e\|}$ provides a smooth decay from 1 to 0, ensuring a continuous and shock-free transition of K between $K_{\max}$ and $K_{\min}$, guaranteeing control stability. From Fig. 3, the factor α impacts the robot's behavior. A large α means high sensitivity with a rapidly $\|e\|$ decaying stiffness curve. A small α means low sensitivity with a gradually decaying curve, tolerating larger errors.

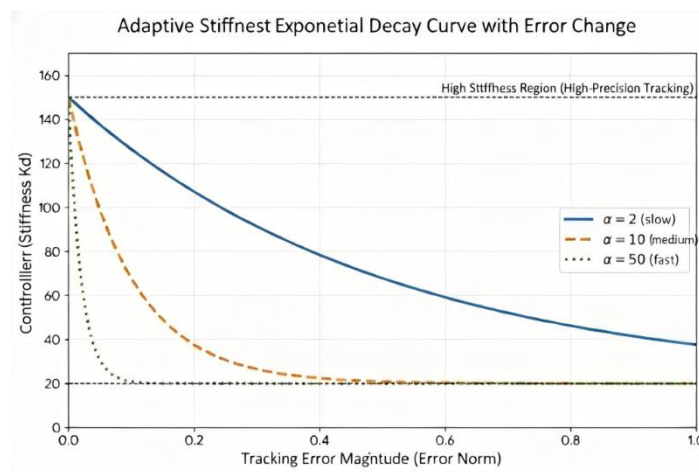


Fig. 3 The influence of different values of α on stiffness changes (Original)

3.2.3 Simulation and analysis in different situations

(a) Simulation of a Sudden Patient Muscle Spasm

Fig. 4 shows the adaptive variation of the controller's stiffness parameter K_d during the process. In the nominal training phase ($t = 1 - 2.8s$), K_d it is kept at a high level (approx. 140 N/m) for high

trajectory tracking accuracy. At $t \approx 5.2$ s when a spasm occurs, the controller detects the large, instantaneous tracking error, identifies it as an unexpected and potentially hazardous human-robot interaction. Then, the safety mechanism activates, and the stiffness K_d is rapidly reduced to a predefined minimum safety threshold (approx. 20 N/m), transitioning the system from high-stiffness training to high-compliance safety mode.

Fig. 5 demonstrates the physical effect of the active compliance strategy. At $t \approx 5.2$ s during the severe spasm, the robot's actual end-effector trajectory (solid blue line) complies with the external force, deviating significantly from the desired trajectory, especially along the Z-axis. This compliant response is important to protect patient safety because it avoids a hard - on - hard conflict and limits the effects of a secondary injury.

Once the simulated spasm subsides ($t > 5.5$ s), when the system gets back to a steady state. The stiffness parameter slowly gets to the nominal level (Fig. 4) reconstructing the high-precision training mode. In the meantime, the real course (Fig. 5) converts to the desirable one without any overshoot.

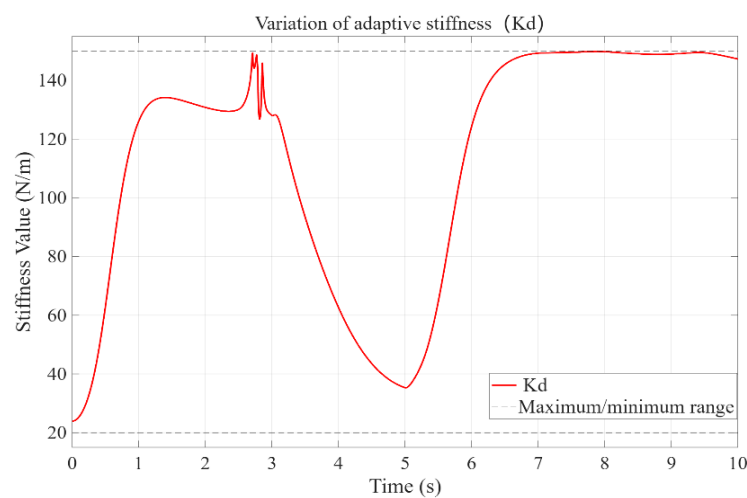


Fig. 4 Changes in stiffness values (Original)

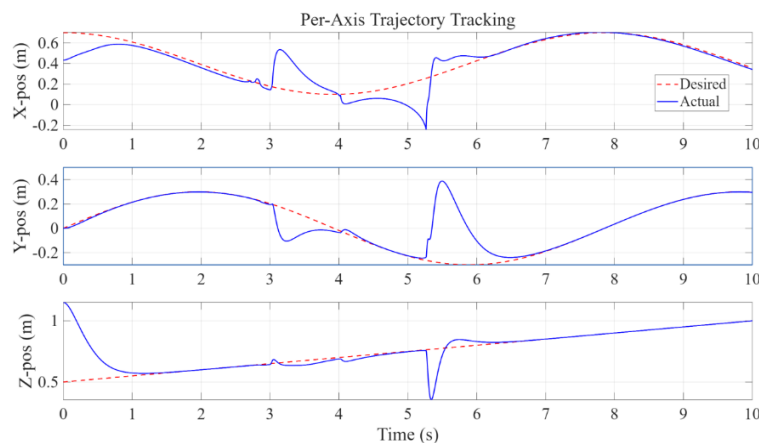


Fig. 5 Comparison of expected trajectory and real trajectory on x-axis, y-axis, and z-axis (Original)

(b) Simulation of force-guided rehabilitation (e.g., active rehabilitation)

Fig. 6 shows the adaptive variation of the controller's stiffness parameter K_d during this process. During training ($t = 0 - 2.8$ s) to allow the controller to be accurate in tracking the trajectory, the stiffness K_d when identifying the errors of the controller was maintained relatively high (approximately 140 N/m). At the stage the participant began active deviation at $t = 3.0$ s, and the controller was able to observe the active tracking error that had developed, and was non-convergent and significant, indicating that the participant did not intend to engage with the scanning process proactively. After that, an active training mechanism was activated and the stiffness K_d was quickly brought to a predetermined minimum aid level (approximately 35 N/m) and the system switches to high-compliance incentive-waiting state.

The effect of the active force withdrawal strategy on the physical level is visibly presented in Fig. 7. The real end-effector trajectory of the robot at the active deviation period ($t = 3.0\text{s} - 5.5\text{s}$) was tolerated in respect to the intended trajectory. This give-up practice plays a significant role in active rehabilitation and serves to discourage the performance of the robot in strict competition with the passive action of the patient and passive dragging dependency, forcing the participant to redirect his/her attention and become an active restorer of correct movement.

After the participant resumed active following ($t > 5.5\text{s}$), the system showed excellent assistive recovery. The stiffness K_d quickly increased to its maximum saturation value (Fig. 6), providing strong guidance for the participant's "active return". Meanwhile, the actual trajectory (Fig. 7) smoothly and rapidly reconverged to the desired trajectory.

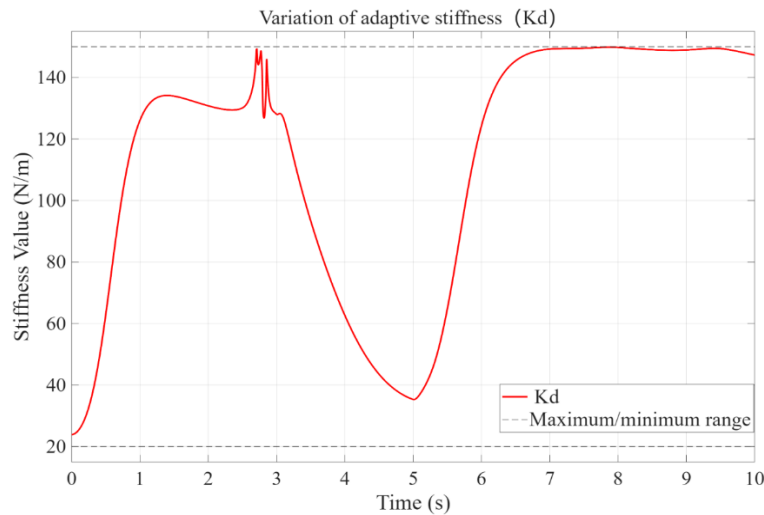


Fig. 6 Changes in stiffness values (Original)

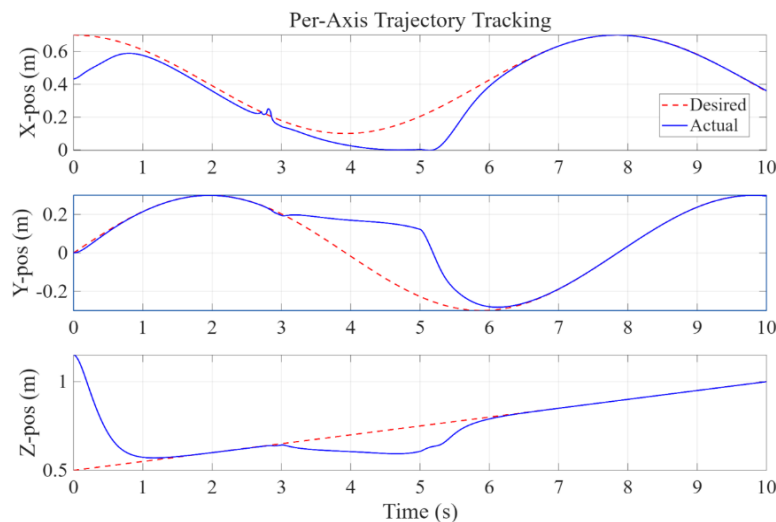


Fig. 7 Comparison of expected trajectory and real trajectory on x-axis, y-axis, and z-axis (Original)

(c) Simulation of individualized Joint Range of Motion (ROM) constraints

Fig. 8 shows the adaptive variation of the controller's stiffness parameter K_d during the process. In the safe region of the trajectory ($t = 0 - 2.8\text{s}$), the controller increased stiffness K_d to about 140 N/m upon detecting tracking errors for training precision. At $t \approx 3.0\text{s}$, when the desired trajectory exceeded the ROM boundary, the controller detected an anticipated error due to ROM limitation. Then, its flexible boundary mechanism was activated, K_d modulating the stiffness to a medium compliant level (approx. 65 - 85 N/m) instead of reducing it to the minimum.

Fig. 9 shows the physical effect of the compliant boundary strategy. From $t \approx 3.0\text{s} - 5.5\text{s}$, the robot's actual end - effector trajectory (blue solid line) was not forced towards the out - of - ROM desired

trajectory (red dashed line), but smoothly clipped along the ROM boundary. This compliant following behavior ensures personalized safety by preventing the robot from forcing the patient's joint beyond the tolerable range. It acts like an elastic wall to limit out - of - range motion and offer continuous guidance, preventing task interruption.

After the desired trajectory re-entered the safe ROM ($t > 5.5s$), the system showed a smooth task resumption characteristic. The stiffness K_d rapidly increased to the maximum (Fig. 8), restoring high-precision tracking. Meanwhile, the actual trajectory (Fig. 9) quickly reconverged to the desired trajectory without overshoot.

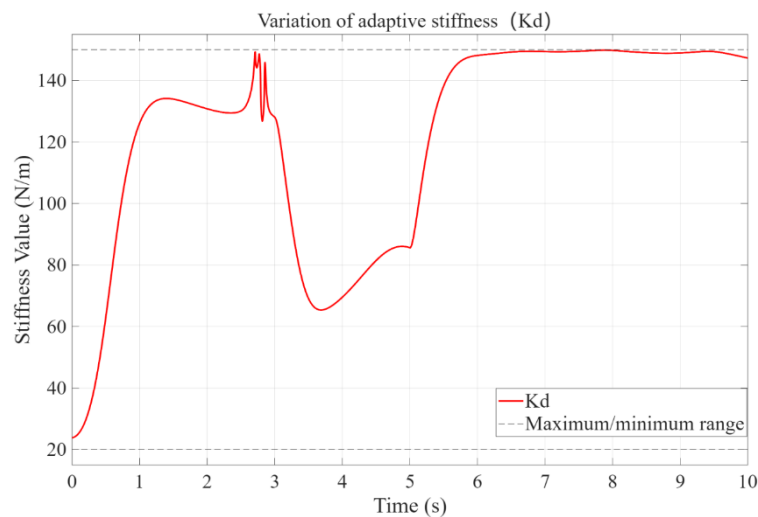


Fig. 8 Changes in stiffness values (Original)

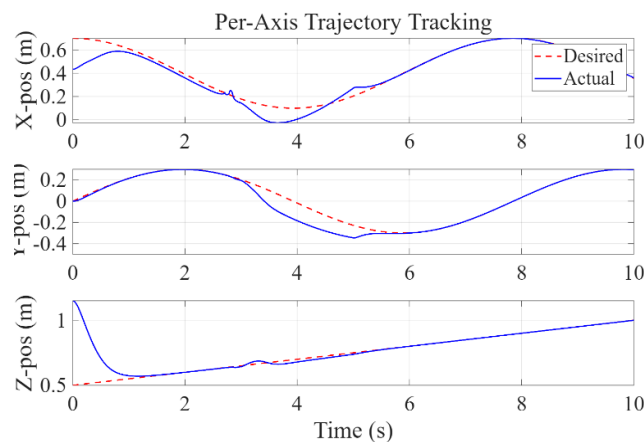


Fig. 9 Comparison of expected trajectory and real trajectory on x-axis, y-axis, and z-axis (Original)

4. Discussion

Although the adaptive impedance control strategy proposed in this paper demonstrates commendable compliance and safety in simulations, it possesses two key limitations regarding deep intelligence and personalization, which constitute the primary directions for future work.

First, the current control strategy remains at a static level in terms of personalization and calibration. Specifically, the controller's core parameters, such as the upper and lower bounds for stiffness adjustment, rely on pre-setting by a therapist before the training commences. This static personalization is incapable of responding in real-time to the patient's immediate state during the training process. Indicatively, in cases where a patient is showing symptoms of exhaustion (e.g., reduced movement velocity, reduced forces of interaction) or gradual regeneration of muscle strength, the system is not capable of dynamically changing the degrees of support or resistance online therefore it is not truly dynamically adaptive [4]. The future research ought to be based on the creation of algorithms that do not only automatically tune initial parameters using the dynamics of baseline

clinical assessment data of a patient (e.g., Fugl-Meyer scores), but also online update the control targets basing in the real-time performance indicators. This would bring the much-needed breakthrough between the traditional personalization that is static and dynamic adaptation.

Second, the safety mechanism of the system is compliant, which, in other words, is passive-reactive. This implies that stiffness reduction or compliant response is activated only after a large interaction force or position error has been detected [3]. Although this model of a post-collision response is not able to prevent injury, it can help to reduce the forces to limit the risk of injury in a much shorter delay time, especially when the events, such as sudden patient muscle spasms, are of high velocity. Rather, it provides compensation only when an impact has already taken place, which suggests that significant changes in the future movie are possible when large amounts of data about the interactions between humans and robots are collected and processed [5]. Thus, the critical aspect of the future consists in utilizing machine learning models (e.g. LSTM networks) by means of gathering and encoding extensive data regarding the interaction between humans and robots. This would transform the system to the proactive-predictive paradigm, as opposed to the passive-reactive one, and would allow it to lessen stiffness prior to a hazardous event happening, which would consequently allow it to reach a much higher level of active safety.

5. Conclusion

More commonly applied to industrial robotics and earlier-generation rehab, traditional fixed-parameter impedance control is employed. The idea behind it is to pre-select virtual parameters such as stiffness, damping, and inertia, to maintain a fixed intensity of forces so that their force-displacement relationship does not change. It is easy to apply and when the conditions along with the task objectives are known and remain constant.

It is however limited as can be seen in neurorehabilitation. The motor abilities of a patient change during the recovery and within a session. The fixed-parameter scheme is not able to adjust to these dynamics, and a discrepancy exists between the intensity of the training and the patient and the growing risk of injuries, which will impact the safety and effectiveness of rehabilitation.

Conversely, the adaptive impedance control suggested in the present research can change the virtual stiffness and damping in real-time depending on the motor performance accompanied by the feedback of the interaction force of the patient. It allows such functions as assist - as - needed, fatigue adaptation, and a smooth manner of switching modes in line with the latest rehabilitation demands of safety, personalization, and progress. As an illustration, it may impose low stiffness during the initial recovery phase and resistance at a later time.

It is an adaptive process which enhances training safety and effectiveness and may be used to assess the progress of the patient towards recovery by monitoring changes in the impedance parameters. Adaptive impedance control is a promising technology in the future of upper-limb rehabilitation robots, and has a higher application value than fixed impedance controls.

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This paper addresses the safety and naturalness issues of human-computer interaction in upper limb rehabilitation robots used in the rehabilitation training of patients with nerve injuries. It proposes an adaptive impedance control method to achieve compliant robot control in response to patient intentions and external disturbances. The authors first establish a robot dynamics model, derive the adaptive impedance control law, and adjust control parameters in real time to cope with changes in interaction forces. Subsequently, multi-scenario simulations validate the effectiveness of the controller. The results show that this method not only ensures the stability and accuracy of trajectory tracking but also significantly reduces the interaction forces applied by the patient, improving compliance performance. Overall, the paper has a clear structure, a reasonable research approach, and sufficient simulation results, possessing high theoretical reference value and application potential; direct acceptance is recommended.							